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Discussion of
"ANALYSIS AND TESTS OF A CYLINDRICAL SHELL ROOF MODEL"

by Bruno Thürlimann and Bruce G. Johnston
(Proc. Sep. 434)

G. C. ERNST,[4] M. ASCE.—The problems met in obtaining experimental

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verification of shell theory, particularly for reinforced concrete shells, are numerous and not easily solved. Aside from the difficulty of interpreting the results of tests in the light of a relatively complex theory, tests on shells are costly and time-consuming. The authors are to be commended for the sustained care demonstrated in the performance of their tests and in the interpretation and presentation of the results of their investigation. In view of a possible objection to the use of a steel model to represent the elastic action of a reinforced concrete structure, data are presented herewith from tests on small reinforced concrete shells made at the University of Nebraska at Lincoln. Although the tests were on long shells rather than short ones, the linear relationship between deflection and load prior to initial cracking provides justification for the authors' use of an elastic material such as steel.

The tests performed at the University of Nebraska were made on three long cylindrical shells of reinforced Portland cement mortar, of such size as to permit testing under the controlled loading of a testing machine. The shells were simply supported at the corners, with a longitudinal span of 60 in. and a tranverse span of 40 in. Each shell had an inside radius of 30 in. and the thickness was uniform longitudinally, but varied transversely from $\frac{3}{8}$ in. at the crown to $\frac{5}{8}$ in. at the edge beams. The reinforcement in the shell consisted of a double layer of 1-in. by 1-in. by 15-gage welded wire mesh in the central 36 in. and a single layer of the same mesh in the outer 12 in. of each end. The double layer in the central part served as over-reinforcement against transverse-bending failure in the regions of greatest shear difference. This shear was developed under vertical loads uniformly distributed in the transverse direction at the third points by specially constructed, multiple-spring loading heads. The primary purpose of the tests was an investigation of the action of long shells at ultimate loads producing high shearing stresses. The longitudinal steel in the edge beams differed for each shell. Data taken on each shell were directed toward ultimate-load analysis, but only that pertinent to the authors' subject is presented herein.

Fig. 20 provides information on the range of virtually elastic action of reinforced concrete shells. The lateral deflections of the shells were entirely consistent (in direction and order of magnitude) with elastic theory, up to the load at which initial diagonal-tension cracking developed. The average vertical deflection at the center of the two edge beams showed a similar characteristic, with the customary early lag in deflection prior to tensile cracking at the level of the longitudinal steel in the edge beams.

With regard to the stability of the structure, two basic approaches must be considered. Design should be based either on (*a*) structural proportions that will assure the full development of the concrete in compression under longi-

tudinal and transverse thrusts at the ultimate capacity of the structure, or (*b*) the use of design stresses that are a conservative reduction from the values obtained from the inelastic buckling theory (20)[4a](21)(22).

[4a] Numerals in parentheses, thus (20) refer to corresponding items in the Bibliography at the end of the discussions.

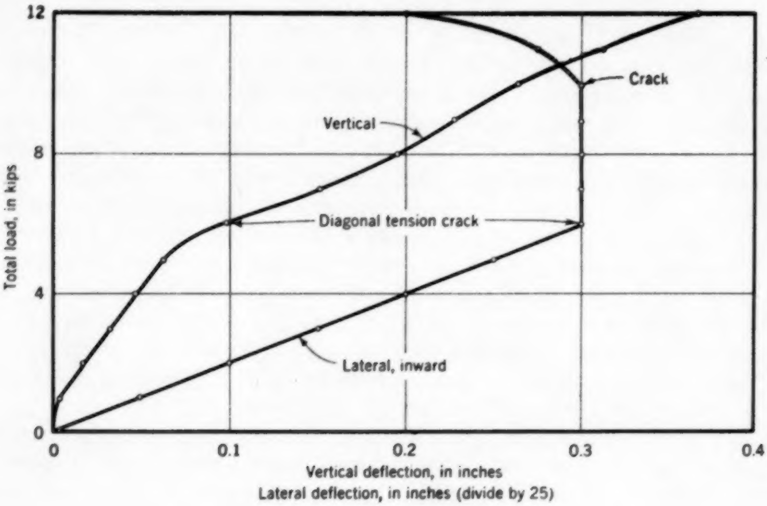

FIG. 20.—EXPERIMENTAL DEFLECTION CURVES

CHARLES S. WHITNEY,[5] M. ASCE.—That there is satisfactory agreement

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between model-test results and theoretical analysis based on the elastic theory has been clearly demonstrated by the authors. That being true, the authors' investigation has added nothing new to the knowledge of the behavior of cylindrical shells stiffened by arch ribs.

Messrs. Thürlimann and Johnston have, however, chosen as part of their report to draw conclusions which are unwarranted either on the basis of the tests or the elastic theory. These conclusions relate to the effects of volume changes which were discussed by the writer in 1950 (19). The comments made under the heading, "Some Special Problems," seem to indicate a lack of understanding of the total effects of the different factors involved. This understanding can only be obtained by performing complete designs on a comparative basis. For this reason, these statements regarding the effects of the various factors are misleading.

Under the heading, "Foundation Movements: Conclusions Concerning Foundation Movements," it is stated that for given movements the moments and thrusts in the rib are proportional to the moment of inertia of the effective section of the rib and further, that the stress in the rib is approximately proportional to the distance from the neutral axis to the fiber being considered. These statements are true according to the elastic theory and were verified by tests on the steel model. The first statement is probably substantially true regarding a concrete structure, but the actual stress condition is completely

different when the shell slab is on the tension side of the neutral axis. Concrete arches must be designed for reversals of moments, and tension in the concrete must be ignored when computing the strength of the section.

Therefore—assuming the same depth of the rib, moving the slab from the center of the rib to the top or bottom approximately doubles the moment of inertia and the moments resulting from abutment rotation without increasing the strength of the section.

Furthermore—a point neglected by the authors—the increased moments caused by the increased stiffness have an important effect on the cost of the foundations, which must be designed for greater eccentricity of thrust.

The same condition exists in regard to volume changes resulting from temperature changes and shrinkage. Bending moments are proportional to the effective stiffness, but the rib strength is not increased by using a T-section when the T-flange is on the tension side.

It is also obvious that, in the case of a difference in temperature between the rib and the shell, higher bending stresses will be produced in the rib if the shell is at the top or bottom rather than at the center of the rib.

The suggestion made by Messrs. Thürlimann and Johnston that the effect of increased stiffness which results from placing the shell at the bottom of the rib can be offset by using a shallower rib to reduce volume-change stresses is impractical because the strength of the rib would at the same time be reduced below that required to carry critical combinations of dead loads and live loads together with the effects of volume changes, abutment movements, and deflection stresses. Deflection stresses are easily controlled without excessive stiffness.

The writer challenges the accuracy of the authors' statements regarding cross bending stress which lead to Eq. 48. The writer has shown (19) that the cross bending moments in the shell produced by placing the shell at the bottom of the rib instead of at the center can be of the same magnitude as those produced by both dead loads and live loads. Their importance cannot be rejected.

To summarize, moving the shell from the bottom of the rib to the center reduces its stiffness by approximately one half and greatly reduces the total bending moments without reducing its strength to resist moments which produce tension at the bottom of the rib. Conversely, placing the shell at the bottom of the rib instead of at the center approximately doubles the rib stiffness and consequently doubles the moments caused by volume change and abutment deflection without any corresponding increase in strength. Placing the shell at the bottom of the rib materially increases the costs of the structure if the structure is consistently designed. The importance of the various factors can be judged only by considering simultaneously their total effect rather than by attempting a separate analysis of their individual effects as the authors have done.

Boyd G. Anderson.[6]—The comments made by the authors under the heading, "Some Special Problems: Location of the Shell with Respect to the Rib," are based on the assumption of an elastic section and are restricted to a

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comparison of rib sections in which the area of the rib is determined solely from the stiffness of the member. Mr. Whitney also dealt with the effect of rib stiffness but he included the characteristics of the concrete members and considered the other interrelated factors occurring under actual design conditions (19)(23).

Long-span arch members are subject to stresses caused by a combination of external loads, volumetric effects, and deflections. The member in all cases must be strong enough to resist all these applied forces. The forces will be at a minimum if the arch rib is flexible enough to limit stresses caused by volume changes, yet stiff enough to limit deflection stresses. As these two conditions are opposing design requirements, they must be balanced so as to yield the minimum possible thrust and moments on the rib.

Even if stiffness were the sole consideration governing the size of the rib, as indicated by Messrs. Thürlimann and Johnston, it would be difficult to predict an effectiveness of the shell flange that would be neither too large (which would result in underestimation of the deflection stresses) nor too small (which would result in underestimation of the volumetric stresses). Furthermore, in this prediction of rib stiffness one would have to consider that the flange is subject to tension over part of its length and compression over the remainder, and that these areas of opposite moment sign would change under different loading combinations.

Rib stiffness, however, is only one of the many factors that determine the total moment and thrust that must be carried by the arch rib. The capacity of the rib to resist these forces is not independent of the sectional area and the lever arm, as was assumed by the authors. Instead, for any given concrete strength the capacity of the member will depend directly on the area of the member, the lever arm of the steel, and the percentage of reinforcing steel. The practical areas of reinforcement will in turn depend (in general) on the area of the rib and (in particular) on the width of the rib. The flange adds little to the capacity of the rectangular section in any case because of limitations in the amount of reinforcement that can be crowded into a rib and it adds nothing at all to the capacity in the part of the arch length where the flanged side is in tension. Because of a possible variation in moment sign, tension in the flange can occur anywhere along the length of the arch.

The authors also state that cross bending of the shell will not be critical because the neutral axis will be near the flange—thus resulting in a low fiber stress in the flange and consequently in a low cross bending effect in the shell. However, although the distribution of moments can be approximated by elastic analysis, the stress in the extreme fibers of reinforced concrete members cannot be determined from the section modulus, as indicated by the authors. If the member is designed efficiently, the concrete in the flange will be at the maximum stress. Cross bending in the shell (which is a function of this fiber stress in the rib) accordingly will usually be an important design consideration.

It is impossible to substitute a small-flanged member for a larger rectangular member and expect that the strength of the smaller section of presumed equal stiffness will be equal to the strength of the larger unflanged section. For any particular section of a long-span arch a certain rib of specific area, reinforcement, and lever arm will be required to carry internal moments and

thrust which will vary in sign and magnitude. Moving the flange to the edge of this rib will not appreciably increase the strength of the member and so will not permit a substantial reduction in size. Moving the flange to the edge, however, will materially increase the stiffness, as noted by Messrs. Thürlimann and Johnston, and it will also increase the volumetric moments and the required strength.

BRUNO THÜRLIMANN[7] AND BRUCE G. JOHNSTON,[8] M. ASCE.—In Fig. 20 Mr. Ernst has shown that thin-shell models made of cement mortar behave elastically up to the cracking load.

The comments of Mr. Anderson and Mr. Whitney are critical of the same points and thus might logically be considered together. Mr. Anderson repeats the statement made under the heading, "Some Special Problems: Stability of the Structure," that the ribs present two opposing design requirements. On the one hand, the stiffness should be large enough to minimize the secondary bending moments that are caused by the deformations under load. Some additional stiffness is required because plastic flow increases these deformations; this important problem has been discussed previously (17) (18). On the other hand, an increase in stiffness has the undesirable effect of increasing the moments caused by volumetric changes resulting from foundation movement, temperature change, or shrinkage. The purpose of the design should be to keep the stiffness at the minimum value required by considerations of stability, secondary moments produced by deformations, and plastic flow under load. It was demonstrated by the tests that, for an elastic material, stiffness and strength of a T-section are both improved when the shell is placed at the edge of the rib. Such a rib may be considerably reduced in size as compared with a rib of equivalent stiffness and strength with the shell attached at the mid-height of the rib. It is true that reinforced concrete sections present a somewhat different problem. Both Mr. Anderson and Mr. Whitney agree that placing the shell either at the top or bottom of the rib increases the stiffness. Stiffness is required all along the ribs to provide sufficient stability. However, there are only a few places—such as at the springline—where the full strength is needed. The strength of the section required to carry a normal force and bending moment increases only if the shell is in compression. Where then should this shell be attached? It is agreed that the total cost of comparative designs is the final answer to the problem. From this point of view Mr. Whitney is convinced that attaching the shell to the middle is the proper solution (19). However, a review of the discussions of his paper (19) and of another (24) will reveal that quite a few other engineers disagree with this conclusion. The writers have not made cost analyses and therefore they cannot enter into this discussion.

Mr. Whitney contends that "* * * the authors' investigation has added nothing new to the knowledge of the behavior of cylindrical shells stiffened by arch ribs."

It is true that the primary contribution is one of experimental confirmation

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of the theory. However, in Fig. 4 is provided a simple, practical procedure by which engineers can determine the effective width of shell that can be assumed to act as part of the arch rib. The tests confirm the use of this effective width; this information was not previously available in such simple form.

Mr. Whitney questions the accuracy of the statements which lead to Eq. 48. As far as elastic behavior is concerned, one can easily prove the statements that confirm Eq. 48. Because Mr. Whitney offered no computations to substantiate his disbelief, such proof is not offered herewith. The conclusion, however, follows from the fact that, under the most extreme conditions, the stress σ_x produced by cross-bending is never more than 173% of the lower fiber stress of the rib. The test results fully confirm this fact which was also derived from theoretical considerations. A short computation made by use of the experimental results for N_ϕ and M_x from Fig. 17, or from other test results (14), will verify this statement.

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(19) **"Cost of Long-Span Concrete Shell Roofs,"** by C. S. Whitney, *Proceedings*, A.C.I., Vol. 21, 1950, p. 765.

(20) **"Inelastic Buckling of Plain and Reinforced Concrete Columns, Plates, and Shells,"** by G. C. Ernst, J. J. Hromadik, and A. R. Riveland, *Bulletin No. 3*, Univ. of Nebraska Eng. Experiment Station, Lincoln, Nebr., August, 1953.

(21) **"Stability of Thin-Shelled Structures,"** by G. C. Ernst, *Journal*, A.C.I., December, 1952, p. 277.

(22) **Discussion** by Herbert A. Sawyer, Jr., of **"Stability of Thin-Shelled Structures,"** by G. C. Ernst, *ibid.*, December, 1953, p. 292-1.

(23) **"Plain and Reinforced Concrete Arches,"** Report of Committee 312, *Proceedings*, A.C.I., Vol. 22, 1951, p. 681.

(24) **Discussion of "Long Span Concrete Arch Hangar at Limestone Air Force Base,"** by Anton Tedesko, *Proceedings*, A.C.I., December, 1950, p. 416.

Discussion of
"SAFETY AND THE PROBABILITY OF STRUCTURAL FAILURE"

by A. M. Freudenthal
(Proc. Sep. 468)

A. CHIBARO,[1] M. ASCE.—While one of the most significant approaches to the application of the laws of probability to the safety of structures has been presented in this paper, the most valuable contribution, in the writer's opinion, is that rational design has at long last received thorough logical scrutiny. How much of the design procedure suggested by the author will be adopted by the Profession in the next decade or two is open to question. Because of this paper, however, engineers have gained a better understanding of the laws governing some of the most important variables involved in engineering problems. Furthermore, they have again been reminded that no "single line of investigation" can produce a solution of a design problem, and that the elastic theory and the plastic theory each in itself is useful in expressing a state of a structure under different intensities of load. It can be concluded from the author's exposition that, because of the random variations pertaining to values of loads and strengths the factor of safety which engineers have been used to consider as a definite number in fact varies. It is a minimum in case the incidence of the maximum service load coincides with the minimum strength of the structure and is a maximum in case the service load is minimum while the strength of the structure is maximum. The probability of occurrence of the most unsafe condition may be considered as a measure of safety.

Although the author's approach may prove to be a milestone in engineering design, we probably are still a long way from the day when the use of the concept "factor of safety" will be supplanted by the use of the concept "probability of failure." However, as forcibly illustrated by the author these concepts are evidently mutually dependent and the latter is a better measure of safety than the former. But in order to use the concept of "probability of failure" a great mass of statistical data pertaining to load effects and strengths must be obtained and correlated. This would appear to be a time consuming and difficult task, especially insofar as load effects are concerned. As the author points out, the frequency distribution pertaining to strength of concrete manufactured under good control conditions is quite different from that pertaining to concrete manufactured with inadequate control. The distribution pertaining to the latter will ordinarily be skewed to a marked degree towards low strengths.

An important factor which needs considerable study, is definition of terms. What are the exact meanings of unserviceability and failure? Does failure begin where serviceability ends? If so, the terms "failure" and "unserviceability" would appear to be synonymous. Such terms found in the author's text as, "service load," "standard load," "standard failure load," "shakedown load," "plastic collapse load," "elastic limit load," "critical design load,"

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"maximum service load," etc., etc., should be clearly defined in simple language.

A large question which finds no satisfactory answer in the writer's mind is: how much nearer are we towards a balanced design procedure by using the laws of probability if a good part (maybe the majority) of the variables are non-random and baffle mathematical analysis? Designers must still be guided largely by horse sense and depend upon their ability to guess correctly.

In the preparation of the subject paper the author has done the Profession a real and much needed service. This service might well be extended by, in his closing discussion, giving definite numerical examples worked out in detail pertaining to Section 12 "Analysis of Failure and Survival: Safety Factors."

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c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.